# The Voters' Curses

## Why We Need Goldilocks Voters

## Supplemental Appendix: Additional proofs and welfare analysis

# B Additional Proofs

We formally prove the claims that a decrease in the voter's cost of paying attention to the campaign can decrease the voter's attention and equilibrium welfare.

**Lemma B.1.** *There exist non-empty open sets of policy costs $\mathcal{K}^G$ and $\underline{\beta}^G \in [0,1)$ such that for all $\beta \in (\underline{\beta}^G, 1)$, there exists a non-empty open set $\mathcal{G}^{\beta^G} \subset [0,1]$ such that the voter's expected equilibrium welfare is lower under $\breve{C}_v(\cdot) \equiv \beta C_v(\cdot)$ than $C_v(\cdot)$ for all $G \in \mathcal{G}^{\beta^G}$.*

*Proof.* When the communication cost function is $\beta C_v(x)$, the voter's level of attention and candidates' communication efforts solve $C_v'(x) = q(1-q)\frac{G}{\beta}y(c)$ and $C'(y(c)) = \frac{1-k_c}{2}x$. A decrease in the communication cost function is thus equivalent to an increase in the gain from change $G$. We know there exists a non-empty open set of policy costs such that an increase in $G$ can decrease the voter's welfare (Proposition 3). Denote this set $\mathcal{K}^G$. Suppose there exists $G^h \in [\underline{G}, \overline{G})$ such that $\forall G > \overline{G}$, the voter's expected equilibrium welfare satisfies $V_v(G) < V_v(G^h)$. Then denote $\underline{\beta}^G = 0$ and for all $\beta \in (0,1)$, the claim holds for $\mathcal{G}^{\beta^G} = (\max\{G^h, \beta\overline{G}\}, \overline{G})$. Suppose there is no such $G^h$. For all $G \in [\underline{G}, \overline{G}]$, define the function

$\phi : [\underline{G}, \overline{G}] \to (\overline{G}, 1)$ as $\phi(G) = \min\ \{Z \in (\overline{G}, 1) \mid V_v(G) = V_v(Z)\}$. Define also $\underline{\beta}^G = \max_{G \in [\underline{G}, \overline{G}]} \frac{G}{\phi(G)}$. By Proposition 3, $\underline{\beta}^G < 1$. And the claim holds true for $\mathcal{G}^{\beta^G} = (\underline{\beta}^G \overline{G}, \overline{G})$.

□

**Lemma B.2.** *There exist non-empty open sets of policy costs $\mathcal{K}^x$ and $\underline{\beta}^x \in [0, 1)$ such that for all $\beta \in (\underline{\beta}^x, 1)$, there exists a non-empty open set $\mathcal{G}^{\beta^x} \subset \mathbb{R}_+$ such that the voter's attention is lower under $\breve{C}_v(\cdot)$ than $C_v(\cdot)$ for all $G \in \mathcal{G}^{\beta^x}$.*

*Proof.* Using Proposition 4 and a similar reasoning as in Lemma B.1, we can show that there exists $\underline{\beta}^x \in [0, 1)$ and $\mathcal{K}^x$ such that the claim holds true for $\beta \in (\underline{\beta}^x, 1)$, $(k_n, k_c) \in \mathcal{K}^x$, and $G \in \mathcal{G}^{\beta^x} = (G_l, \overline{G})$, where $G_l$ is a lower bound satisfying $G_l < \overline{G}$ such that for all $G \in (G_l, \overline{G})$ the voter's attention is lower under $\breve{C}_v(\cdot)$ than $C_v(\cdot)$. □

**Proposition B.1.** *Suppose that the voter's cost of communication decreases from $C_v(\cdot)$ to $\beta C_v(\cdot)$, with $\beta < 1$. There exist non-empty open sets of policy costs, $\beta$ and gain from change $G$ such that the voter's expected equilibrium welfare and level of attention are lower with $\beta C_v(\cdot)$ than $C_v(\cdot)$.*

*Proof.* Using Lemmas B.1 and B.2, there exist an open set of policy costs $\mathcal{K}^G \cap \mathcal{K}^x$ (from Propositions 3 and 4, one can check that the intersection is not empty) and $\underline{\beta} = \max\{\underline{\beta}^G, \underline{\beta}^x\} \in [0, 1)$ such that the claim holds true for the non-empty open sets $\mathcal{K}^G \cap \mathcal{K}^x$, $(\underline{\beta}, 1)$, and $\mathcal{G}^\beta = \mathcal{G}^{\beta^G} \cap \mathcal{G}^{\beta^x}$ (which is non empty by Lemmas B.1 and B.2). □

# C Ranking assessments

In what follows, we show that there exists $\breve{k}_n : (0, 1)^2 \to (0, 1)$ such that for all $(G, k_c) \in (0, 1)^2$ and $k_n \leq \breve{k}_n(G, k_c)$ a separating assessment (where each candidate chooses $p = 1$ only if competent and $p = 0$ otherwise) maximizes the voter's ex-ante welfare. In this section, we simply compare the voter's ex-ante welfare associated with different assessments, without proving whether they are can be part of an equilibrium, but we exclude assessments that cannot be equilibria. Our result naturally implies that when $k_n \leq \breve{k}_n(G, k_c)$, if a separating equilibrium exists, it maximizes the voter's expected equilibrium welfare. In what follows, we denote the voter's (ex-ante) expected welfare in an assessment when a type $t \in \{c, n\}$ candidate $j \in \{1, 2\}$ plays strategy $p_j(t) \in \{0, 1\}$ by $V_v((p_1(c), p_1(n)), (p_2(c), p_2(n)))$.

Denote $\alpha^* = y^*(c)x^*$ the probability that a competent candidate $j$'s campaign is successful (voter observes $p_j(c) = 1$) in a separating assessment. We have:

$$V_v((1,0),(1,0)) = q^2G + q(1-q)(1+\alpha^*)G - C_v(x^*) = qG + q(1-q)\alpha^*G - C_v(x^*) \quad \text{(C.1)}$$

From Lemma 2, $q(1-q)\alpha^*G > C_v(x^*)$ since the voter maximizes her expected utility at the communication subform and $x^* > 0$ (by assumption $C_v(0) = 0$). We thus have:

$$V_v((1,0),(1,0)) > qG \quad \text{(C.2)}$$

(C.2) directly implies that $V_v((1,0),(1,0)) > V_v((0,0),(0,0)) = 0, \ \forall G > 0$.

An assessment in which candidate $j \in \{1, 2\}$ separates and candidate $-j$ does not separate cannot be an equilibrium so it is excluded from the analysis.[1] (This assessment would lead

[1]Suppose it is. The voter's expected payoff from electing candidate $-j$ is 0, from electing candidate $j$ is strictly positive (independent of the communication outcome). So the voter always elects candidate $j$ and candidate $j$ does not need to exert communication effort (since successful communication has no effect on his probability of winning). But by Lemma 1,

to a worse expected payoff than a separating assessment for the voter).

Consider now an assessment in which (wlog) candidate 1 pools on the new policy ($p_1(c) = p_1(n) = 1$) and candidate 2 pools on the status quo policy ($p_2(c) = p_2(n) = 0$). Denote by $x^{p_1}$ the voter's level of attention (notice $y_2^{p_1}(t) = 0,\ \forall t \in \{c, n\}$); and by $\alpha_1^{p_1}(c)$ and $\alpha_1^{p_1}(n)$ the probability that communication is successful with a competent and non-competent candidate 1, respectively. A necessary condition for such an assessment to be an equilibrium is that the voter elects candidate 1 when communication is successful and elects candidate 2 when it is not successful.[2] The expected utility of the voter is then:

$$V_v((1,1),(0,0)) = q\alpha_1^{p_1}(c)G + (1-q)\alpha_1^{p_1}(n)L - C_v(x^{p_1}) \tag{C.3}$$

Since $\alpha_1^{p_1}(c) < 1$ ($C'(1) = C_v'(1) = 1$), we have $V_v((1,1),(0,0)) < qG + (1-q)\alpha^{p_1}(n)L - C_v(x^{p_1}) < qG$. From (C.2), it must be that the voter is better off in a separating assessment.

We now show that a separating equilibrium gives a higher expected welfare to the voter than an assessment in which both candidates commit to the new policy independent of their type. The voter elects candidate $j$ if communication with $j$ is successful and not successful with his opponent. The voter tosses a fair coin when communication with both candidates is successful and not successful.[3] Denote by $x^p$ the voter's attention in a pooling assessment.

proposing $p_j(c) = 0$ is a profitable deviation for a competent candidate $j$. Hence, we have reached a contradiction.

[2] Otherwise, we can reach a contradiction using a similar reasoning as in footnote 1.

[3] The result holds cases when the voter listens more to one candidate and does not randomize when communication with both candidates is successful or unsuccessful.

Also denote by $\alpha_j^p(t)$ the probability that communication is successful with a type $t \in \{c, n\}$ candidate $j \in \{1, 2\}$. We can show that $\alpha_1^p(t) = \alpha_2^p(t) \equiv \alpha^p(t),\ t \in \{c, n\}$.[4] Imposing the symmetry, the voter's expected utility is then:

$$\begin{aligned} V_v((1,1),(1,1)) &= q^2 G + (1-q)^2 L + 2q(1-q)\Big(\alpha^p(c)(1-\alpha^p(n))G + \alpha^p(c)\alpha^p(n)\frac{G+L}{2} \\ &\quad +(1-\alpha^p(c))(1-\alpha^p(n))\frac{G+L}{2} + (1-\alpha^p(c))\alpha^p(n)L\Big) - C_v(x^p) \\ &= q^2 G + (1-q)^2 L + q(1-q)\Big((1+\alpha^p(c)-\alpha^p(n))G \\ &\quad + (1-\alpha^p(c)+\alpha^p(n))L\Big) - C_v(x^p) \end{aligned} \tag{C.4}$$

Since $-L/G > q/(1-q)$, we have that $q^2 G + q(1-q)((\alpha^p(c) - \alpha^p(n))G + (1 - \alpha^p(c) + \alpha^p(n))L) - C_v(x^p)$ is a strict upper bound for $V_v((1,1),(1,1))$. As a consequence, the difference $V_v((1,0),(1,0)) - V_v((1,1),(1,1))$ must be strictly larger than

$$q(1-q)(G-L)(1-\alpha^p(c)+\alpha^p(n)) + C_v(x^p) + q(1-q)G\alpha^* - C_v(x^*) > 0$$

The last inequality follows from $\alpha^p(c) < 1$ and (C.2).

Lastly, we show that a separating assessment gives the voter a higher expected payoff than an asymmetric assessment in which candidate 1 pools on $p = 1$ ($p_1(c) = p_1(n) = 1$) and candidate 2 separates ($p_2(c) = 1$ and $p_2(n) = 0$) for $k_n \leq \breve{k}_n(G, k_c)$. Denote by $x^a$ the voter's attention in this asymmetric equilibrium. Denote by $\alpha_j^a(t) = x^a y_j^a(t)$ the probability that

[4] The reasoning is the same as in Lemma 2. Note however that the welfare-maximizing level of communication may not be the highest solution to the system of equation that defines $x^p$ and a competent and non-competent candidates $j$'s communication efforts.

communication is successful with a type $t \in \{c, n\}$ candidate $j \in \{1, 2\}$ ($y_j^a(t),\ j \in \{1, 2\}$ is a type $t \in \{c, n\}$ candidate $j$'s communication effort), with $\alpha_2^a(n) = 0$ by Lemma 1. We first establish some properties of the asymmetric assessment. First, the voter elects candidate 1 only if communication with candidate 1 is successful and communication with candidate 2 is not successful since $-L/G > q/(1-q)$. The expected utility of the voter after rearranging is:

$$V_v((1,1),(1,0)) = q^2G + q(1-q)\Big[G(1+\alpha_1^a(c)) + (L-G)\alpha_1^a(n)(1-\alpha_2^a(c))\Big] + (1-q)^2\alpha_1^a(n)L - C_v(x^a) \tag{C.5}$$

Observe that by always electing candidate 2, the voter gets in expectation $qG$. Hence a necessary condition for this assessment to be an equilibrium is (notice that condition (C.6) is equivalent to saying that the voter prefers to elect candidate 1 than candidate 2 after successful communication with candidate 1 only):

$$qGy_1^a(c) + (1-q)y_1^a(n)L + q(L-G)y_1^a(n)(1-\alpha_2^a(c)) \geq 0 \tag{C.6}$$

Denote $V_j^a(p_j, y_j; t)$ the expected utility of a candidate $j$ of type $t$ in a separating assessment. Supposing (C.6) holds, candidate 1 is elected only if his communication with the voter is successful, but his opponent is not. We thus have:

$$V_1^a(1, y_1^a(t); t) = (q(1-\alpha_2^a(c)) + (1-q))\alpha_1^a(t)(1-k_t) - C(y_1^a(t)),\ t \in \{c, n\} \tag{C.7}$$

$$V_2^a(1, y_2^a(c); c) = [q(1-\alpha_1^a(c) + \alpha_1^a(c)\alpha_2^a(c)) + (1-q)(1-\alpha_1^a(n) + \alpha_1^a(n)\alpha_2^a(c))](1-k_t) - C(y_2^a(c)) \tag{C.8}$$

Using (C.5)-(C.8), voter's attention and candidates' communication efforts in an asymmetric equilibrium (supposing it exists) satisfy:

$$C_v'(x^a) = q(1-q)Gy_1^a(c) + (1-q)^2 Ly_1^a(n) + q(1-q)(L-G)(1-2\alpha_2^a(c))y_1^a(n) \tag{C.9}$$

$$C'(y_1^a(t)) = (q(1-\alpha_2^a(c)) + (1-q))x^a(1-k_t) \tag{C.10}$$

$$C'(y_2^a(c)) = (q\alpha_1^a(c) + (1-q)\alpha_1^a(n))x^a(1-k_c) \tag{C.11}$$

From (C.10), as $k_n \to k_c$, $y_1^a(n) \to y_1^a(c)$ and (C.6) does not hold. So an asymmetric equilibrium does not exist. Suppose that $y_1^a(n)$ decreases with $k_n$, whereas voter's attention and the other communication efforts increase with $k_n$ then there exists $\acute{k}_n^a(G,k_c) \in (k_c, 1)$ such that condition C.6) is satisfied if and only if $k_n \geq \acute{k}_n^a(G,k_c)$ (notice that the threshold $\acute{k}_n^a(k_c,G)$ would increase if the voter's attention and other communication efforts decrease with $k_n$).[5] Notice that when (C.6) holds with equality, $x^a > 0$ by (C.9). We now compare the voter's welfare in a separating and an asymmetric assessments. We have: $V_v((1,0),(1,0)) \geq q^2G + q(1-q)G\left(1 + y^*(c)x^a\right) - C_v(x^a)$, by definition of $x^*$. Therefore, we have that the difference $V_v((1,0),(1,0)) - V_v((1,1),(1,0))$ is weakly larger than

$$(1-q)x^a\left[qGy^*(c) - qGy_1^a(c) - (1-q)y_1^a(n)L - q(L-G)y_1^a(n)(1-\alpha_2^a(c))\right] \tag{C.12}$$

Assume $qGy_1^a(c)+(1-q)y_1^a(n)L+q(L-G)y_1^a(n)(1-\alpha_2^a(c))$ is strictly increasing with $k_n$ (the analysis can be easily adapted when this is not always satisfied). We have that $y^*(c) > 0$, and it does not depend on $k_n$, by Lemma 2. At $k_n = \acute{k}_n^a(G,k_c)$, we have $V_v((1,0),(1,0)) - V_v((1,1),(1,0)) > 0$ by (C.6). Therefore, there exists: $\tilde{k}_n(G,k_c) \in (\acute{k}_n^a(G,k_c), 1]$ such that the

[5](C.6) is always satisfied as $k_n \to 1$ since $y_1^a(n) \to 0$. So $\acute{k}_n^a(G,k_c) < 1$.

term between brackets on the left-hand side of (C.12) is positive whenever $k_n \leq \tilde{k_n}(G, k_c)$.[6] This implies that there exists $\breve{k_n}(G, k_c) \geq \tilde{k_n}(G, k_c)$ (with strict inequality if $x^* \neq x^a$ and $\tilde{k_n}(G, k_c) < 1$) such that $V_v((1,0),(1,0)) \geq V_v((1,1),(1,0))$, $\forall k_n \leq \breve{k_n}(G, k_c)$.[7]

[6]Notice that under our assumption that $qGy_1^a(c)+(1-q)y_1^a(n)L+q(L-G)y_1^a(n)(1-\alpha_2^a(c))$ is strictly increasing with $k_n$, $\tilde{k_n}^a(G, k_c) < 1$ if and only if $y_1^a(c) > y^*(c)$ evaluated at $k_n = 1$. This in turn requires $q(1 - \alpha_2^a(c)) + (1 - q) > 1/2$. See footnote 7 in Appendix A for more details.

[7]We assume that the asymmetric equilibrium exists when $k_n = \breve{k_n}(G, k_c)$. This is not guaranteed since we have not checked candidates' incentive compatibility constraints. However, this simply implies that the upper bound on $k_n$ is *less* tight than suggested in the analysis.