The Voters' Curses

Why We Need Goldilocks Voters

Supplemental Appendix: Extensions

D Introducing asymmetry between candidates

We capture the idea that voters might be significantly more informed about incumbent's competence level and platforms than they are about challengers in a simple environment. An incumbent (for simplicity, candidate 1) relies on his policy record, which gives the voter an expected payoff of $R \in (qG, G)$, unless the economy is hit by a large negative unforeseen contingency. The unforeseen contingency leads to the incumbent's removal with probability one,[1] and we denote the probability of this 'tail event' by ψ. The payoffs associated with the two types of challenger are as in the baseline model. As such, in the absence of unforeseen contingencies and without additional information from the campaign, the voter prefers the incumbent to the challenger ($R > qG$). Two parameters capture the notion and strength of incumbency advantage: R (value of an incumbent to the voter) and $1 - \psi$ (how safe the incumbent is). In this environment, the voter's expected payoff from a separating assessment

[1]This assumption allows to introduce, in the simplest possible way, uncertainty in the electoral outcome that is necessary for a separating equilibrium to exists. See the recent literature on natural disasters (e.g., Healy and Malhotra, 2009) for support to this assumption.

is given by

$$q\alpha_2(c)G + q(1 - \alpha_2(c)(\psi G + (1 - \psi)R) + (1 - q)\psi R$$

The voter elects the challenger under two conditions. First, communication with the challenger (candidate 2) is successful (she then gets a payoff of G). Second, communication with the challenger is unsuccessful, but there is a negative shock which affects the incumbent (she then gets an expected payoff of $(1 - q\alpha_2(c))G$). The two types of challenger's payoff are, respectively, $(1 - k_c)(xy_2(c) + (1 - xy_2(c))\psi) - C(y_2(c))$ and ψ. The voter's attention and competent candidate's communication effort then solve the following system:

$$C'(y_2(c)) = (1 - k_c)(1 - \psi)x \tag{D.1}$$

$$C_v'(x) = q(1 - \psi)(G - R)y_2(c) \tag{D.2}$$

Substituting back into the candidates' payoffs, it is easy to verify that (i) the communication subgame has a unique strictly positive solution $(y_2^*(c), x^*)$, and (ii) the incentive compatibility conditions (IC) are given by

$$(1 - k_c)x^* y_2^*(c)(1 - \psi) - C(y_2^*(c)) \geq \psi$$

$$(1 - k_n)x^* \hat{y}_2(n)(1 - \psi) - C(\hat{y}_2(n)) \leq \psi.$$

where $\hat{y}_2(n)$ solves $C'(y) = (1 - k_n)(1 - \psi)x^*$. Changes in G affect the incentive compatibility constraints of the two types in the same way as the baseline model (see Lemma A.5). Interestingly, the two parameters associated with the strength of the incumbent affect the equilibrium in different ways: a lower likelihood of negative shock (ψ) increases electoral communication and improves democratic responsiveness, while a higher baseline payoff (R)

from the incumbent has the opposite effect.

E Multiple voters: Common knowledge model

Let $\mathcal{N} = \{1, ..., N+1\}$ denote the set of voters. We assume that each voter $i \in \mathcal{N}$ can either vote for candidate $j \in \{1, 2\}$ ($e_i = j$) or abstain ($e_i = \emptyset$). When indifferent, voters toss a fair coin as before. Candidate $j \in \{1, 2\}$ is elected with probability 1 if he receives strictly more votes than his opponent $-j$: $|\{i \in \mathcal{N} : e_i = j\}| > |\{i \in \mathcal{N} : e_i = -j\}|$ In case of equality (i.e, $|\{i \in \mathcal{N} : e_i = j\}| = |\{i \in \mathcal{N} : e_i = -j\}|$), we suppose that the election is decided by a fair coin toss. In particular, notice that if all voters abstain, candidate j wins with probability 1/2. The utility of voter $i \in \mathcal{N}$ is:

$$u_i(p_e, x_i) = \begin{cases} G_i p_e - C_v(x_i) \text{ if } e \text{ is competent} \\ \\ L_i p_e - C_v(x_i) \text{ if } e \text{ is non-competent} \end{cases} \tag{E.1}$$

We assume $G_i = \lambda_i G$ and $L_i = \lambda_i L$, with $qG + (1-q)L < 0$ and $\lambda_i \in [\underline{\lambda}, 1]$, with $\underline{\lambda} \in (0, 1)$, for all $i \in \mathcal{N}$. Using a similar reasoning as in Feddersen and Pesendorfer (1996), it can be checked that it is always a weakly dominated strategy for an uninformed voter to abstain (since the electorate is balanced). Hence, we assume thereafter that an uninformed voter always abstains. The utility of a politician is unchanged.

Each citizen collects information about candidates' platform independently, but voters are able to freely pool their information before the election. As a result, if at the voting stage one voter is informed, then all other voters are informed. We use the subscript CK to denote equilibrium actions in this setting. The timing is:

1. Nature draws the candidates' type: $t_j \in \{c, n\},\ j \in \{1, 2\}$.

2. Candidate $j \in \{1, 2\}$ observes (only) his type and chooses a platform: the status quo policy ($p_j = 0$), or the new policy ($p_j = 1$).

3. The electoral campaign takes place. Candidates 1 and 2 choose their communication efforts y_1, y_2, respectively. Each voter $i \in \{1, ..., N+1\}$ chooses her level of attention x_i. With probability $P_j = 1 - \prod_{i=1}^{N+1}(1 - x_i y_j))$, communication is successful and voters observes candidate j's platform. With probability $1 - P_j$, voters do not learn p_j.

4. Voters vote for one of the two candidates ($e_i \in \{1, 2\}$) or abstain ($e_i = \emptyset$).

5. The elected candidate e implements p_e and payoffs are realized.

Robustness of Proposition 2. We begin by showing that the key insights of the baseline model are robust to this more general environment. For ease of exposition, we assume $\lambda_1 = 1$.

Lemma E.1. *Suppose a separating equilibrium exists. Non-competent candidates exert no effort ($y^*_{CK}(n) = 0$). Voter i's level of attention and competent candidates' communication efforts are defined by the following system of equation:*

$$C'(y^*_{CK}(c)) = \frac{1 - k_c}{2} \sum_{i=1}^{N+1} x^*_{CK}(G_i) \prod_{l \neq i}(1 - x^*_{CK}(G_l) y^*_{CK}(c)) \tag{E.2}$$

$$C'_v(x^*_{CK}(G_i)) = q(1 - q) y^*_{CK}(c) G_i \prod_{l \neq i}(1 - x^*_{CK}(G_l) y^*_{CK}(c)),\ \forall i \in \mathcal{N} \tag{E.3}$$

Proof. Lemma A.3 holds in this setting. We focus on a competent candidate 1 without loss of generality. Voters observe p_1 with probability P_1. In that case, candidate 1 wins with probability $q\Big[(1 - P_2) + P_2/2\Big] + (1 - q)$. When his opponent is competent, probability q, candidate 1 wins with probability $1/2$ when voters learn candidate 2's platform and probability 1 otherwise.[2] When his opponent is non-competent, he wins with probability 1. When voters do not learn candidate 1's platform, candidate 1 is elected with probability $(q(1-P_2)+(1-q))/2$: He is elected with probability 1/2 when voters do not learn his opponent platform. Rearranging, a competent candidate 1 maximizes with respect to y_1:

$$EU_1(1,y_1;c) = \frac{1-qP_2+P_1}{2}(1-k_c) - C(y_1) \tag{E.4}$$

Using $y_1(n) = y_2(n) = 0$ (this is without loss of generality) and a similar reasoning as in the main appendix, voter i's expected utility is then:

$$EU_i(x_i) = q(1-q)\left[P_1G_i+(1-P_1)\frac{G_i}{2}\right]+(1-q)q\left[P_2G_i+(1-P_2)\frac{G_i}{2}\right]+q^2G_i-C_v(x_i) \tag{E.5}$$

The probability that voters learn candidate 1's platform is: $P_1 = 1-\prod_{i=1}^{N+1}(1-x_iy_1(c))$. As a result, the FOC for candidate 1 and the voter are, respectively, $C'(y_1(c)) = \frac{1-k_c}{2}\sum_{i=1}^{N+1}x_i\prod_{l\neq i}(1-x_ly_1(c))$ and $C_v'(x_i) = q(1-q)\prod_{l\neq i}(1-x_ly_1(c))\frac{G}{2}y_1(c)+q(1-q)\prod_{l\neq i}(1-x_ly_2(c))\frac{G}{2}y_2(c)$. Combining the two equations yields the system (notice $y_1(c) = y_2(c)$). □

Lemma E.2. *In any communication subgame, it must be that* $\sum_{l=1}^{N+1}\alpha_l\prod_{h\neq l}(1-\alpha_h)\leq 1$

Proof. Let $H(\alpha_1,...,\alpha_{N+1}) = \sum_{l=1}^{N+1}\alpha_l\prod_{h\neq l}(1-\alpha_h)$ and suppose, contrary to the statement, that $H>1$. Notice that H is linear in every α_i. Therefore, it must be that $H(\alpha_1,...,\alpha_{N+1})\leq$

[2] When voters are indifferent, they randomize between both candidates. Therefore, the probability candidate 1 is elected is 1/2. To see that, assume N is even. Politician j wins if he gets $N/2+1$ votes or more. Since the number of votes (denoted υ) he obtains is a binomial distribution, the probability he gets k votes is: $P(\upsilon = k) = \binom{N+1}{k}\frac{1}{2^{N+1}}$. The probability he gets $N/2+1$ votes or more is: $P(\upsilon \geq N/2+1) = 1/2$ due to the symmetry. When N is odd, the probability candidate j is elected is $P(\upsilon \geq (N+1)/2+1)+P(\upsilon = (N+1)/2)*(1/2) = 1/2$.

$\max\{H(0,\alpha_2,...,\alpha_{N+1}), H(1,\alpha_2,...,\alpha_{N+1})\}$. Since $H(1,\alpha_2,...,\alpha_{N+1}) = \prod_{h\neq 1}(1-\alpha_h) < 1$, it must be that $H(\alpha_1,\alpha_2,...,\alpha_{N+1}) \leq H(0,\alpha_2,...,\alpha_{N+1})$. By the same reasoning, for every $i \in \mathcal{N}$, it must be that $H(0,...,0,\alpha_i,...,\alpha_{N+1}) \leq H(0,...,0,\alpha_{i+1},...,\alpha_{N+1})$. Hence, we must have $H(\alpha_1,...,\alpha_{N+1}) \leq H(0,...,0,\alpha_{N+1}) = \alpha_{N+1} \leq 1$, a contradiction. □

Lemma E.3. *There exists a strictly positive solution to the system of equations (E.2)-(E.3).*

Proof. Denote by $\alpha_{CK}(G_i) = y^*_{CK}(c)x^*_{CK}(G_i)$ the probability of successful communication between voter $i \in \mathcal{N}$ and one of the two politicians. The system (E.2)-(E.3) can be re-expressed as

$$\alpha_{CK}(G_i)^{\gamma-1} = q(1-q)\frac{1-k_c}{2}G\lambda_i \prod_{l\neq i}(1-\alpha_{CK}(G_l)) \sum_{l=1}^{N+1} \alpha_{CK}(G_l) \prod_{h\neq l}(1-\alpha_{CK}(G_h)),\ \forall i \in \mathcal{N} \tag{E.6}$$

The proof proceeds in three steps. First, we show that $\alpha_{CK}(G_i)$ is bounded above by $\lambda_i(1-1/\gamma)$ for all $i \in \mathcal{N}$. Second, we show that for all $i \in \mathcal{N}$, there is one-to-one mapping between $\alpha_{CK}(G_1)$ and $\alpha_{CK}(G_i)$. Third, we prove existence of a solution to (E.6).

Step 1. Notice that (i) $\prod_{l\neq 1}(1-\alpha_{CK}(G_l)) \sum_{l=1}^{N+1} \alpha_{CK}(G_l) \prod_{h\neq l}(1-\alpha_{CK}(G_h)) < 1$, by Lemma E.2, and (ii) $q(1-q) \leq 1/4$. Hence $\alpha_{CK}(G_1) < \left(\frac{1}{4}\right)^{\frac{1}{\gamma-1}}$. Since the difference $\left(\frac{1}{4}\right)^{\frac{1}{\gamma-1}} - 1 + \frac{1}{\gamma}$ converges to zero from below as $\gamma \to +\infty$, $\alpha_{CK}(G_1) < 1 - 1/\gamma$ for all finite γ. By a similar reasoning, we get: $\alpha_{CK}(G_1) < \lambda_i(1-1/\gamma)$ for all $i \in \mathcal{N}$.

Step 2. By taking the ratio of $\alpha_{CK}(G_i)$ and $\alpha_{CK}(G_1)$, we obtain $\alpha_{CK}(G_i)^{\gamma-1}(1-\alpha_{CK}(G_i)) - \lambda_i\alpha_{CK}(G_1)^{\gamma-1}(1-\alpha_{CK}(G_1)) = 0$. The smallest solution of the equation (the other, being above $1-1/\gamma$, contradicts the FOC since $\alpha_{CK}(G_i) < \lambda_i(1-1/\gamma)$ from step 1) implicitly defines a one-to-one mapping $A_i(\alpha_{CK}(G_1)) : [0, 1-1/\gamma) \to [0, 1-1/\gamma)$, that is strictly increasing and such that $A_i(0) = 0$ and $A_i(\alpha) < \alpha$ for all i such that $\lambda_i < 1$.

Step 3. Using step 2, we can re-express the FOC of voter 1 as a fixed point (using the notation $\alpha_{CK}(G_1) \equiv \alpha_1$ for ease of exposition):

$$H_1(\alpha_1) = q(1-q)\frac{1-k_c}{2}G\prod_{l\neq 1}(1-A_l(\alpha_1))\sum_{l=1}^{N+1}A_l(\alpha_1)\prod_{h\neq l}(1-A_h(\alpha_1)) - \alpha_1^{\gamma-1} \tag{E.7}$$

H_1 must have a strictly positive fixed point, since (i) $H_1(0) = 0$, (ii) $H_1(1-1/\gamma) < 1/4 - (1-1/\gamma)^{\gamma-1} < 0$, and (iii) $H_1'(0) > 0$. To see why the latter must hold, notice first that $H_1'(0) = q(1-q)\frac{1-k_c}{2}G[1+\sum_{l=2}^{N+1}A_l'(0)]$. Suppose now that $A_l'(0) < 0$. By continuity of $A_l(\cdot)$, there exist a value $\epsilon > 0$ such that $A_l(\epsilon) < 0$, which contradicts the fact that $A_l(\alpha_1) > 0$, $\forall \alpha_1 > 0$. Hence $1+\sum_{l=2}^{N+1}A_l'(0) > 0$ and $H_1'(0) > 0$. Once $\alpha_{CK}(G_1)$ is determined, one obtains $y_{CK}^*(c)$ from $Y(\alpha_1) = \left[q(1-q)\frac{1-k_c}{2}\sum_{l=1}^{N+1}A_l(\alpha_1)\prod_{h\neq l}(1-A_h(\alpha_1))\right]^{\frac{1}{\gamma}}$, and the remaining values from the other FOCs. □

Let $\mathcal{S}_{CK}$ denote the set of solutions of (E.7). Each solution generates a set of equilibrium probabilities. Let $\alpha_{CK}^- = \inf \mathcal{S}_{CK}$ and $\alpha_{CK}^+ = \sup \mathcal{S}_{CK}$. The following Lemma shows how these quantities vary with G.

Lemma E.4. *The equilibrium communication probabilities α_{CK}^- and α_{CK}^+ are strictly increasing with G.*

Proof. We prove the Lemma for α_{CK}^-, the argument for α_+^* is analogous. Since $H_1(0) = 0$, $H_1(1-1/\gamma) < 0$ and $H_1'(0) > 0$, it must be that at α_-^*, $H_1(\cdot)$ crosses the horizontal axis from above. Hence, $H_1'(\alpha_-^*) < 0$ (and $H_1'(\alpha_+^*) < 0$). Since $\frac{\partial H_1}{\partial G} > 0$, applying the Implicit Function Theorem to $H_1(\alpha_-^*) = 0$ yields $\frac{\partial \alpha_-^*}{\partial G} = -\frac{\partial H_1/\partial G}{\partial H_1/\partial a_-^*} > 0$. □

Denote $y_{CK}^-(c)$ the equilibrium communication effort and $x_{CK}^-(G_i)$ (for all $i \in \mathcal{N}$) the level of attention implicitly defined respectively by (E.2) and (E.3) when the equilibrium communication probability is α^-_{CK}. Similarly, denote $y^+_{CK}(c)$ and $x^+_{CK}(G_i)$ the equilibrium choices implicitly defined respectively by (E.2) and (E.3) when the equilibrium communication probability is α^+_{CK}.

Lemma E.5. *$x^-_{CK}(G_i)$ and $x^+_{CK}(G_i)$ are strictly increasing with G for all $i \in \mathcal{N}$.*

Proof. We show the result for $x^-_{CK}(G_i)$, the reasoning can be extended to $x^+_{CK}(G_i)$. Using a similar reasoning as in Lemma E.3, we can express all $x^-_{CK}(G_i)$ as a one-to-one mapping: $X^-_{CK}(x^-_{CK}(G_1)) : [0, 1-1/\gamma] \to [0, 1-1/\gamma]$. This implies $\text{sgn}(\partial x^-_{CK}(G_1)/\partial G) = \text{sgn}(\partial x^-_{CK}(G_i)/\partial G)$ for all $i \in \mathcal{N}$ (remember $G_i = \lambda_i G$ for all i).

The proof then proceeds by contradiction. Suppose $\partial x^-_{CK}(G_1)/\partial G \leq 0$. Then using (E.2), we get:

$$(\gamma-1)(y^-_{CK}(c))^{\gamma-2}\frac{\partial y^-_{CK}(c)}{\partial G} = \frac{1-k_c}{2}\sum_{i=1}^{N+1}\frac{\partial x^-_{CK}(G_i)}{\partial G}\prod_{l\neq i}^{N+1}(1-A_l(\alpha^-_{CK}(G_i))) + \\ + \frac{1-k_c}{2}\sum_{i=1}^{N+1}x^-_{CK}(G_i)\sum_{l\neq i}^{N+1}\left(-\frac{\partial A_l(\alpha^-_{CK})}{\partial G}\right)\prod_{m\neq i,l}^{N+1}(1-A_m(\alpha^-_{CK})) \tag{E.8}$$

Given $-\frac{\partial A_l(\alpha^-_{CK})}{\partial G} < 0$ by Lemmas E.3 and E.4, this implies that $\frac{\partial y^-_{CK}(c)}{\partial G} < 0$. But then α^-_{CK} is strictly decreasing in G, a contradiction. □

To show that any separating equilibrium would disappear when G is large (for a non-empty open set of parameter values), we focus on the equilibrium with the lowest probability of successful communication $\alpha^-_{CK}(G_1)$, and define $\alpha^-_{CK}(G_i) \equiv A_i(\alpha^-_{CK}(G_1))$. When a competent candidate deviates and proposes the status quo policy, he does not invest in communication and is elected with probability $1/2$ whenever communication is unsuccessful with his opponent (which occurs with probability $1-q(1-\prod_{i=1}^{N+1}(1-\alpha^-_{CK}(G_i)))$. Using the reasoning in Lemma

E.1, the incentive compatibility constraint (IC) of a competent candidate is:

$$\begin{bmatrix} 1 - q(1 - \prod_{i=1}^{N+1}(1 - \alpha_{CK}^{-}(G_i)))+ \\ +(1 - \prod_{i=1}^{N+1}(1 - \alpha_{CK}^{-}(G_i))) \end{bmatrix} \frac{1 - k_c}{2} - C(y_{CK}^{-}(c)) \geq \frac{1 - q(1 - \prod_{i=1}^{N+1}(1 - \alpha_{CK}^{-}(G_i)))}{2} \tag{E.9}$$

Conversely, when a non-competent candidate deviates and proposes the new policy, his communication effort is defined by the following equation (using a similar reasoning as in Lemma E.1):

$$C'(\hat{y}_{CK}^{-}(n)) = \frac{1 - k_n}{2} \sum_{i=1}^{N+1} x_{CK}^{-}(G_i) \prod_{l \neq i}(1 - x_{CK}^{-}(G_l)\hat{y}_{CK}^{-}(n)) \tag{E.10}$$

The (IC) of a non-competent candidate is then:

$$\begin{bmatrix} 1 - q(1 - \prod_{i=1}^{N+1}(1 - \alpha_{CK}^{-}(G_i)))+ \\ +(1 - \prod_{i=1}^{N+1}(1 - x_{CK}^{-}(G_i)\hat{y}_{CK}^{-}(n)) \end{bmatrix} \frac{1 - k_n}{2} - C(\hat{y}_{CK}^{-}(n)) \leq \frac{1 - q(1 - \prod_{i=1}^{N+1}(1 - \alpha_{CK}^{-}(G_i)))}{2} \tag{E.11}$$

Proposition E.1. *There exists an open non-empty set of policy costs (k_c, k_n) such that for all $N \geq 0$, N finite, there exist unique $\underline{G}(N) > 0$ and $\overline{G}(N) \in (\underline{G}(N), 1)$ such that a separating equilibrium exists if and only if the voter's gain from change is in an intermediate range:*

$$\underline{G}(N) \leq G \leq \overline{G}(N)$$

Proof. We omit the dependence on N whenever possible for ease of exposition. First, start with a competent candidate. Denote $V(1, y_{CK}^{*}(c); k_c, G)$ and $V(0, 0; k_c, G)$ the expected utilities of a competent candidate from choosing the new policy ($p = 1$) and associated equilibrium communication level and from choosing the status quo policy ($p = 0$) as a function of k_c, respectively (the left-hand side and right-hand side of (E.9), respectively). Define the set $\mathcal{K}_c^{CK}(N) = \{k_c \in (0, 1) : V(1, y^*_{CK}; k_c, 1) = V(0, 0; k_c, 1)\}$. This set is non-empty since $V(1, y^*_{CK}; 0, 1) > V(0, 0; 0, 1)$, $V(1, y^*_{CK}; 1, 1) < V(0, 0; 1, 1)$, and the expected utilities are continuous in k_c (x^*_{CK} and $y^*_{CK}(c)$ are continuous in k_c). Define $\overline{k_c}^{CK}(N) = \min \mathcal{K}_c^{CK}(N)$.[3] By continuity in k_c, it must be that $\overline{k_c}^{CK}(N) > 0$. Lemmas E.4 and E.5 imply that an increase in the voter's gain from reform G relaxes the (IC) of a competent candidate. Therefore using a similar reasoning as in Proposition 2 for all $k_c \in (0, \overline{k_c}^{CK}(N))$, there exists a unique $\underline{G}(N) \in (0, 1)$ such that a competent type's strategy is incentive compatible if and only if $G \geq \underline{G}(N)$.[4]

Consider now a non-competent candidate. Denote $V(1, \hat{y}_{CK}(n); k_n, G)$ and $V(0, 0; k_n, G)$ the expected utilities of a non-competent candidate from choosing the new policy ($p = 1$) and associated equilibrium communication level and from choosing the status quo policy ($p = 0$) as a function of k_n, respectively (the left-hand side and right-hand side of (E.11), respectively). $V(1, \hat{y}_{CK}(n); k_n, G)$ is strictly decreasing with k_n. Define $\overline{k_n}^{CK}(k_c; N) \in (\overline{k_c}(N), 1)$ the unique solution to $V(1, \hat{y}_{CK}(n); k_n, 1) = V(0, 0; k_n, 1)$. Lemmas E.4 and E.5 imply that an increase in the voter's gain from reform G tightens the (IC) of a non-competent candidate. Therefore, we can apply a similar reasoning as in Proposition 2 for all $k_n \in (k_c, \overline{k_n}^{CK}(k_c; N))$ and show that there exists a unique $\overline{G}(N) \in (0, 1)$ such that a non-competent type's strategy is incentive compatible if and only if $G \leq \overline{G}(N)$. The remaining of the proof follows from the same reasoning as Proposition 2. □

The effect of the electorate's size We now focus on how the bounds defining the existence

[3]Notice that unlike Proposition 2, we do not claim that $\mathcal{K}_c^{CK}(N)$ is a singleton.

[4]$\underline{G}(N) > 0$ comes from $V(1, y^*_{CK}; k_c, 0) < V(0, 0; k_c, 0)$ for all $k_c > 0$.

of a separating equilibrium are affected by the number of voters. We start by assuming a fully homogeneous electorate ($\lambda_i = \lambda_j$) $\forall \{i, j\} \in \mathcal{N}^2$). We focus on the case when $\lambda_i = \underline{\lambda}\ \forall i \in \mathcal{N}$ and show two results. First, when the size of the electorate is large enough, an increase in the number of voters mitigates the curse of the uninterested voter (i.e., $N' > N \Rightarrow \underline{G}(N') < \underline{G}(N)$) and exacerbates the curse of the interested voter (i.e., $\overline{G}(N') < \overline{G}(N)$). Second, when the size of the electorate is large enough, the equilibrium bounds are strictly lower than in the baseline model, denoted $N = 0$ ($\underline{G}(N) < \underline{G}(0)$ and $\overline{G}(N) < \overline{G}(0)$).

Lemma E.6. *There exists a unique symmetric solution to the system of equations (E.2)-(E.3)*

Proof. The system (E.2)-(E.3) becomes (omitting superscripts, subscripts, and arguments for simplicity):

$$y^{\gamma-1} =(N+1)(1-xy)^N \frac{1-k_c}{2} x \tag{E.12}$$

$$x^{\gamma-1} =(1-xy)^N q(1-q)G\underline{\lambda} y \tag{E.13}$$

Denote $\alpha = xy$. Multiplying both equations, we get:

$$\alpha^{\gamma-2} = (N+1)(1-\alpha)^{2N} \frac{(1-k_c)}{2} q(1-q)G\underline{\lambda} \tag{E.14}$$

Existence of a solution is guaranteed by Lemma E.1. To see uniqueness, notice that the right-hand side of (E.14) is strictly increasing with α, while the left-hand side of (E.14) is strictly decreasing with α. So there is a unique solution. Denote it by $\underline{\alpha}^*_{CK}$. By Lemma E.1, $y^*_{CK}(c)$ and $x^*_{CK}(\underline{\lambda}G)$ are uniquely determined as a function of α. This prove the claim. □

Lemma E.7. $(1-\underline{\alpha}^*_{CK})^N$ *is strictly decreasing with* N.

Proof. To explicitly account for the dependence on N, let $E(N) = (1 - \underline{\alpha}^*_{CK}(N))^N$. To simplify the analysis, assume that N is a continuous variable (all values are continuous in N). Taking the natural logarithm (ln) of $E(N)$ and differentiating yields

$$\frac{E'(N)}{E(N)} = -N\frac{(\underline{\alpha}^*_{CK})'(N)}{1-\underline{\alpha}^*_{CK}(N)} + \ln(1-\underline{\alpha}^*_{CK}(N)) \tag{E.15}$$

(E.14) can be re-expressed as $\underline{\alpha}^*_{CK}(N)^{\gamma-2} = (N+1)E(N)^2 Z_I$, where $Z_I(\underline{\lambda}) = \frac{(1-k_c)}{2}q(1-q)G\underline{\lambda}$. Taking the ln and differentiating yields

$$(\gamma-2)\frac{(\underline{\alpha}^*_{CK})'(N)}{\underline{\alpha}^*_{CK}(N)} = \frac{1}{N+1} + 2\frac{E'(N)}{E(N)} \tag{E.16}$$

Suppose $E'(N) \geq 0$, then (E.16) implies $(\underline{\alpha}^*_{CK})'(N) > 0$. But by (E.15), this implies that $E'(N) < 0$. Hence we reached a contradiction and $E'(N) < 0$. □

The next lemma shows that the probability that no voter is informed decreases as the size of the electorate increases for N large enough. It also states that this probability is always increasing as the number of voters increases when γ is not too large.

Lemma E.8. *For all $N \geq 0$, there exists $\overline{\gamma}(N) \in (2, \infty)$, with $\overline{\gamma}(0) = 2 - \frac{\ln(Z_I(\underline{\lambda}))}{1/2-\ln(\exp(1/2)-1)} \geq 4.23$, $\overline{\gamma}(\cdot)$ strictly increasing with N, and $\lim_{N\to\infty}\overline{\gamma}(N) = \infty$, such that if $\gamma \leq \overline{\gamma}(N)$, then for all $m > 0$, $(1-\underline{\alpha}^*_{CK}(N))^{N+1} > (1-\underline{\alpha}^*_{CK}(N+m))^{N+m+1}$.*

Proof. To explicitly account for the dependence on N, let $S(N) = (1 - \underline{\alpha}^*_{CK}(N))^{N+1}$. To simplify the analysis, assume that N is a continuous variable (all values are continuous in N). Taking the ln of $S(N)$ and differentiating yields

$$\frac{S'(N)}{S(N)} = -\frac{(N+1)(\underline{\alpha}^*_{CK})'(N)}{1-\underline{\alpha}^*_{CK}(N)} + \ln(1-\underline{\alpha}^*_{CK}(N)) \tag{E.17}$$

Notice that $(\underline{\alpha}^*_{CK})'(N) \geq 0$ directly implies $S'(N) < 0$. Suppose in what follows that $(\underline{\alpha}^*_{CK})'(N) < 0$. $S'(N) \geq 0$ implies that:

$$(\underline{\alpha}^*_{CK})'(N) \leq \frac{\ln(1-\underline{\alpha}^*_{CK}(N))(1-\underline{\alpha}^*_{CK}(N))}{N+1} \tag{E.18}$$

From (E.16), using (E.15), we get:

$$(\gamma-2)\frac{(\underline{\alpha}^*_{CK})'(N)}{\underline{\alpha}^*_{CK}(N)} = \frac{1}{N+1} - \frac{2N(\underline{\alpha}^*_{CK})'(N)}{1-\underline{\alpha}^*_{CK}(N)} + 2\ln(1-\underline{\alpha}^*_{CK}(N)) \tag{E.19}$$

Rearranging yields:

$$(\underline{\alpha}^*_{CK})'(N) = \frac{\frac{1}{N+1} + 2\ln(1-\underline{\alpha}^*_{CK}(N))}{\frac{\gamma-2}{\underline{\alpha}^*_{CK}(N)} + \frac{2N}{1-\underline{\alpha}^*_{CK}(N)}} \tag{E.20}$$

Given $\gamma > 2$ and $\underline{\alpha}^*_{CK}(N) < 1$, we get:

$$(\underline{\alpha}^*_{CK})'(N) > \frac{\ln(1-\underline{\alpha}^*_{CK}(N))(1-\underline{\alpha}^*_{CK}(N))}{N} + \frac{1-\underline{\alpha}^*_{CK}(N)}{2N(N+1)} \tag{E.21}$$

Using (E.18) and (E.21), we can see that a necessary condition for $S'(N) \geq 0$ is:

$$\frac{\ln(1-\underline{\alpha}^*_{CK}(N))(1-\underline{\alpha}^*_{CK}(N))}{N} + \frac{1-\underline{\alpha}^*_{CK}(N)}{2N(N+1)} < \frac{\ln(1-\underline{\alpha}^*_{CK}(N))(1-\underline{\alpha}^*_{CK}(N))}{N+1}$$
$$\Leftrightarrow\ \underline{\alpha}^*_{CK}(N) > 1-\exp(-1/2) \equiv \overline{\alpha} \tag{E.22}$$

Given Lemma E.6, $\underline{\alpha}^*_{CK}(N) > \overline{\alpha}$ if and only if:

$$\overline{\alpha}^{\gamma-2} < (N+1)(1-\overline{\alpha})^{2N} Z_I(\underline{\lambda}), \tag{E.23}$$

with $Z_I(\underline{\lambda}) = \frac{(1-k_c)}{2} q(1-q)G\underline{\lambda} < \frac{1}{8}$. Rearranging (E.23) yields:

$$\gamma > 2 + \frac{N - \ln(N+1) - \ln(Z_I(\underline{\lambda}))}{1/2 - \ln(\exp(1/2) - 1)} \equiv \overline{\gamma}(N) \tag{E.24}$$

Therefore, whenever $\gamma \leq \overline{\gamma}(N)$, we have $S'(N) < 0$ as claimed. It can easily be checked that $\overline{\gamma}(N)$ is strictly increasing with N and $\lim_{N\to\infty} \overline{\gamma}(N) = +\infty$. Finally, the upper bound on $Z_I(\underline{\lambda})$ implies $\overline{\gamma}(0) \geq 2 + \frac{\ln(8)}{1/2-\ln(\exp(1/2)-1)} \approx 4.23$. □

Lemma E.8 implies that, for N large enough, the probability of communication failure is necessarily decreasing in the size of the electorate.

Corollary E.1. *For all $\gamma > 2$, there exists $\overline{N}(\gamma)$, increasing with γ and satisfying $\overline{N}(\gamma) = 0$ for all $\gamma \leq \overline{\gamma}(0)$, such that if $N \geq \overline{N}(\gamma)$, then $(1 - \underline{\alpha}^*_{CK}(N))^{N+1} > (1 - \underline{\alpha}^*_{CK}(N+m))^{N+m+1}$ for all $m > 0$.*

Lemma E.9. *For all $\gamma > 2$, there exists $\widetilde{N}(\gamma)$, decreasing with γ and satisfying $\widetilde{N}(\gamma) = 0$ for all $\gamma \geq \overline{\gamma}(0)$, such that if $N \geq \widetilde{N}(\gamma)$ then for all $m > 0$, $\underline{\alpha}^*_{CK}(N+m) < \underline{\alpha}^*_{CK}(N)$.*

Proof. As before we treat N as a continuous variable. Using (E.20), $(\underline{\alpha}^*_{CK})'(N) \leq 0 \Leftrightarrow \frac{1}{N+1} + 2\ln(1 - \underline{\alpha}^*_{CK}(N)) \leq 0$ or $\underline{\alpha}^*_{CK}(N) \geq 1 - \exp\left(-\frac{1}{2(N+1)}\right) \equiv \tilde{\alpha}(N)$. A sufficient and necessary condition for $\underline{\alpha}^*_{CK}(N) \geq \tilde{\alpha}(N)$ is:

$$\left(1 - \exp\left(-\frac{1}{2(N+1)}\right)\right)^{\gamma-2} \leq (N+1)\exp\left(-\frac{N}{(N+1)}\right) Z_I(\underline{\lambda}) \tag{E.25}$$

The left-hand side is strictly decreasing with N and converges to 0 as N tends to infinity. The right-hand side is strictly increasing with N and converges to ∞ as N tends to ∞. Furthermore, $\left(1 - \exp\left(-\frac{1}{2}\right)\right)^{\gamma-2} \leq Z_I(\underline{\lambda})$ if and only if $\gamma \geq \overline{\gamma}(0)$. This implies that for all $\gamma > \overline{\gamma}(0)$, $(\underline{\alpha}^*_{CK})'(N) < 0$ for all $N \geq 0$ and for all $\gamma \leq \overline{\gamma}(0)$, there exists a unique $\widetilde{N}(\gamma)$ such that $(\underline{\alpha}^*_{CK})'(N) < 0$ for all $N \geq \widetilde{N}(\gamma)$. The comparative statics on $\widetilde{N}(\cdot)$ follows from the fact that the left-hand side of (E.25) is strictly decreasing with γ. Notice that $\lim_{\gamma\to 2} \widetilde{N}(\gamma) < \infty$. □

The following technical Lemma determines a lower bound on $\underline{\alpha}^*_{CK}(N)$.

Lemma E.10. *For all γ and all finite $M > 0$, there exists a finite $\widehat{N}(\gamma; M) > M - 1$, strictly decreasing with γ, such that $\underline{\alpha}^*_{CK}(N) \geq \frac{M}{N+1}$ for all $N \geq \widehat{N}(\gamma; M)$.*

Proof. From Lemma E.6, for all $N \geq M - 1$, $\underline{\alpha}^*_{CK}(N) \geq \frac{M}{N+1}$ if and only if:

$$\left(\frac{M}{N+1}\right)^{\gamma-2} \leq (N+1)\left(1 - \frac{M}{N+1}\right)^{2N} Z_I(\underline{\lambda})$$

taking the ln of both sides, the expression becomes

$$(\gamma - 2)(\ln(M) - \ln(N+1)) \leq 2N\ln(N+1-M) - (2N-1)\ln(N+1) + \ln(Z_I(\underline{\lambda})) \quad \text{(E.26)}$$

Define the function $Q(N; M) = 2N\ln(N+1-M) - (2N-1)\ln(N+1)$. It satisfies: $Q'(N; M) = 2(\ln(N+1-M) - \ln(N+1)) + \frac{2N}{N+1-M} - \frac{2N-1}{N+1}$ and (after rearranging)

$$Q''(N; M) = -\frac{(1 - 6M + 5M^2) + 2(1 - 3M + M^2)N + N^2}{(N+1-M)^2(N+1)^2}$$

This implies that there exists $N^d \geq 0$ such that $Q'(N; M)$ is always decreasing for $N \geq N^d$. Furthermore, $\lim_{N\to\infty} Q'(N; M) = 0$. Therefore, there exists a unique $\acute{N}(M) \geq M$ such that $Q(N; M)$ is strictly increasing for all $N > \acute{N}(M)$. Furthermore, $\lim_{N\to\infty} Q(N) = \infty$.[5]

[5]Rewrite $Q(N)$ as $Q(N) = 2N\ln(1 - \frac{M}{N+1}) + \ln(N+1)$. The first term converges to $-2M$

This implies that the right-hand side of (E.26) is strictly increasing with N for $N \geq \acute{N}(M)$ and tends to ∞ as N tends to ∞. The left-hand side of (E.26) is strictly decreasing with N for $N \geq 0$ and tends to $-\infty$ as N tends to ∞. Hence there exists a unique $\widehat{N}(\gamma; M) \in (M-1, \infty)$ such that for all $N \geq \widehat{N}(\gamma; M)$, the inequality (E.26) is satisfied. Since the left-hand side of (E.26) is strictly decreasing with γ, we have $\widehat{N}(\gamma; M)$ strictly decreasing with γ (by the Implicit Function Theorem). □

The next proposition shows that, in large electorates, an increase in the number of voters mitigates the curse of the uninterested voter and exacerbates the curse of the interested voter.

Proposition E.2. *For all $\gamma > 2$, there exists $\widehat{N}(\gamma) < \infty$, strictly decreasing with γ, such that: If $N \geq \widehat{N}(\gamma)$ and (k_c, k_n) such that $(\underline{G}(N), \overline{G}(N)) \in (0,1)^2$, then for all $m > 0$, $\underline{G}(N+m) < \underline{G}(N)$ and $\overline{G}(N+m) < \overline{G}(N)$.*

Proof. Define the communication efforts and the levels of attention as explicit function of N as for $\underline{\alpha}^*_{CK}(N)$. Since all variables are continuous in N, we assume for simplicity that N is a continuous variable. Using (E.9), for all $N \geq 0$, $\underline{G}(N)$ (assuming it exists) is implicitly defined by:

$$\frac{(1-(1-\underline{\alpha}^*_{CK}(N))^{N+1}))}{2}(1-k_c) - C(y^*_{CK}(N)) = \frac{1-q(1-(1-\underline{\alpha}^*_{CK}(N))^{N+1})}{2}k_c \quad \text{(E.27)}$$

Assume $N \geq \overline{N}(\gamma)$ so the right-hand side is strictly decreasing with N by Corollary E.1. It is thus sufficient to show that the left-hand side of (E.27) is weakly increasing with N. By the Envelope Theorem, this is equivalent to show after rearranging that $R(N) \equiv (N +$

(by L'Hopital's rule), the second term diverges to ∞.

$1)(x^*_{CK})'(N)y^*_{CK}(N) - \ln(1-\underline{\alpha}^*_{CK}(N))(1-\underline{\alpha}^*_{CK}(N)) \geq 0.$[6] Using (E.13, multiplying both sides with by x, taking the ln and differentiating with N yields after rearranging using (E.20):

$$\begin{aligned}\gamma\frac{(x^*_{CK})'(N)}{x^*_{CK}(N)} &= \ln(1-\underline{\alpha}^*_{CK}(N)) - N\frac{(\underline{\alpha}^*_{CK})'(N)}{1-\underline{\alpha}^*_{CK}(N)} + \frac{(\underline{\alpha}^*_{CK})'(N)}{\underline{\alpha}^*_{CK}(N)} \\ &= \frac{\gamma(1-\underline{\alpha}^*_{CK}(N))\ln(1-\underline{\alpha}^*_{CK}) - \frac{((N+1)\underline{\alpha}^*_{CK}(N)-1)}{N+1}}{2[(N+1)\underline{\alpha}^*_{CK}(N)-1]+\gamma(1-\underline{\alpha}^*_{CK}(N))}\end{aligned} \tag{E.28}$$

Plugging (E.28) into the definition of $R(N)$ and rearranging yields:

$$R(N) \propto \begin{bmatrix} -\underline{\alpha}^*_{CK}(N)((N+1)\underline{\alpha}^*_{CK}(N)-1) \\ -\ln(1-\underline{\alpha}^*_{CK})\gamma(N+1)\underline{\alpha}^*_{CK}(N)(1-\underline{\alpha}^*_{CK}(N)) \\ +\ln(1-\underline{\alpha}^*_{CK}(N))(2\gamma-\gamma^2(1-\underline{\alpha}^*_{CK}(N)))(1-\underline{\alpha}^*_{CK}(N)) \end{bmatrix}$$

Notice that $2\gamma - \gamma^2(1-\underline{\alpha}^*_{CK}(N)) \leq 0 \Leftrightarrow \underline{\alpha}^*_{CK}(N) \leq 1-\frac{2}{\gamma}$. Using a similar reasoning as in Lemma E.8, there exists a finite $N^{\gamma 1}$ such that for all $N \geq N^{\gamma 1}$, $\underline{\alpha}^*_{CK}(N) \leq 1-\frac{2}{\gamma}$ (since $\gamma > 2$). Consequently for all $N \geq N^{\gamma 1}$, $\ln(1-\underline{\alpha}^*_{CK}(N))(2\gamma-\gamma^2(1-\underline{\alpha}^*_{CK}(N)))(1-\underline{\alpha}^*_{CK}(N)) \geq 0$. In addition, denote the function $\mathcal{R}(\underline{\alpha}^*_{CK}(N)) = ((N+1)\underline{\alpha}^*_{CK}(N)-1)+\ln(1-\underline{\alpha}^*_{CK}(N))\gamma(N+1)(1-\underline{\alpha}^*_{CK}(N))$. This yields $\mathcal{R}'(\underline{\alpha}^*_{CK}(N)) = (N+1)(-\gamma+1-\gamma\ln(1-\underline{\alpha}^*_{CK}(N)))$. We have $\mathcal{R}'(\underline{\alpha}^*_{CK}(N)) \leq 0 \Leftrightarrow \underline{\alpha}^*_{CK}(N) \leq 1-\exp\left(-(1-\frac{1}{\gamma})\right)$. Using a similar reasoning as in Lemma E.8, there exists a finite $N^{\gamma 2}$ such that for all $N \geq N^{\gamma 2}$, $\underline{\alpha}^*_{CK}(N) \leq 1-\exp\left(-(1-\frac{1}{\gamma})\right)$. By Lemma E.10, there exists $\widehat{N}(\gamma;1)$ such that for all $N \geq \widehat{N}(\gamma;1)$, $\underline{\alpha}^*_{CK}(N) \geq 1/(N+1)$. So for $N \geq \max\{N^{\gamma 2}, \widehat{N}(\gamma;1)\}$, the function $\mathcal{R}(\cdot)$ satisfies: $\mathcal{R}(\underline{\alpha}^*_{CK}(N)) \leq \mathcal{R}(\frac{1}{N+1}) = \ln(1-\frac{1}{N+1})\gamma N < 0$ and consequently, $-\underline{\alpha}^*_{CK}(N)(((N+1)\underline{\alpha}^*_{CK}(N)-1)-\ln(1-\underline{\alpha}^*_{CK})\gamma(N+1)(1-\underline{\alpha}^*_{CK}(N))) \geq 0$.

[6]Notice that we compare $\frac{(1-(1-\underline{\alpha}^*_{CK}(N))^{N+1}))}{2}(1-k_c) - C(y^*_{CK}(N))$ and $\frac{(1-(1-\underline{\alpha}^*_{CK}(N+m))^{N+m+1}))}{2}(1-k_c) - C(y^*_{CK}(N+m))$.

Putting all the results above together, there exists a finite $N^{-}(\gamma) \geq 0$ such that for all $N \geq N^{-}(\gamma)$, the left-hand side of (E.27) is weakly increasing with N. Since by Corollary E.1, the right-hand side of of (E.27) is strictly decreasing with N. There exists $\widehat{N}(\gamma) \leq N^{-}(\gamma)$ such that for all $N \geq \widehat{N}(\gamma)$ we obtain $\underline{G}(N)$ is strictly decreasing with N by the Implicit Function Theorem (since by Proposition E.1, the left-hand side of (E.27 is strictly increasing with G, while the right-hand side is strictly decreasing with G).[7] □

We now show that for very large electorate the curse of the interested voter is worse than in the case of a representative voter (i.e., $\overline{G}(N) < \overline{G}(0)$ for N large enough). To do so, we first consider some asymptotic property of the communication efforts and levels of attention (assuming a separating equilibrium exists).

Lemma E.11. *As N converges to infinity, the following holds:*

(i) $x^*_{CK}(\underline{\lambda}G; N)$ *converges to* 0

(ii) $(1 - \underline{\alpha}^*_{CK}(N))^{N+1}$ *converges to* 0

Proof. We first prove by contradiction that $(1 - \underline{\alpha}^*_{CK}(N))^{N}$ converges to 0 as N converges to infinity. Suppose $E(N)$ (defined in Lemma E.8) converges to $z > 0$ as N converges to infinity. Then, using $\underline{\alpha}^*_{CK}(N)^{\gamma-2} = (N+1)E(N)^2 Z_I(\underline{\lambda})$, it must be that there exists $\overline{N}$ such that $\underline{\alpha}^*_{CK}(N) = 1 \; \forall N > \overline{N}$ and, as a consequence, $E(N) = 0 \; \forall N > \overline{N}$, which contradicts the initial assumption.

The result above coupled with the definition of $x^*_{CK}(\underline{\lambda}G; N)$ (see (E.13)) and the fact that $y^*_{CK}(N) \leq 1$ directly implies point (i). Since $\lim_{N\to\infty} x^*_{CK}(\underline{\lambda}G; N) = 0$ and $y^*_{CK}(N) \leq 1$, we must have $\lim_{N\to\infty} \underline{\alpha}^*_{CK}(N) = 0$. Rewrite $(1 - \underline{\alpha}^*_{CK}(N))^{N+1} = (1 - \underline{\alpha}^*_{CK}(N))^{N} + (1 - \underline{\alpha}^*_{CK}(N))^{N}\underline{\alpha}^*_{CK}(N)$. The results above directly imply point (ii). □

[7] Since $N \in \mathbb{N}$, we cannot guarantee $\widehat{N}(\gamma) < N^{-}(\gamma)$.

Let $y^*_{CK}(0)$ denote a competent candidate's communication effort in the baseline model with $C(y) = y^\gamma/\gamma$ (see Lemma 2). Define similarly a non-competent candidate's communication effort $\hat{y}_{CK}(0)$ and the probability of successful communication: $\alpha^*_{CK}(0)$ and $\hat{\alpha}^*_{CK}(0)$. The following Lemma shows that a competent candidate's communication effort is lower in a large enough electorate than with a single voter.

Lemma E.12. *There exists a finite $N^y \geq 1$ such that for all $N \geq N^y$, then $y^*_{CK}(N) \leq y^*_{CK}(0)$.*

Proof. The proof proceeds in three steps. First, we show that there exists a finite solution $\underline{M}$ to the equation $\left(M \exp(-M)\frac{1-k_c}{2}\right)^{\frac{1}{\gamma-1}} = y^*_{CK}(0)$. Second we show that $y^*_{CK}(N)$ converges to a value strictly lower than $y^*_{CK}(0)$. Lastly, we use a continuity argument to conclude the proof.

Step 1. Denote the function $\overline{Y}(M) = \left(M \exp(-M)\frac{1-k_c}{2}\right)^{\frac{1}{\gamma-1}}$. This function is strictly decreasing with M for $M \geq 1$ and $\lim_{M\to\infty} \overline{Y}(M) = 0$. Given that $y^*_{CK}(0) > 0$ (by Lemma 1), there exists a unique finite $\overline{M} \geq 1$ such that for all $M > \overline{M}$, $\overline{Y}(M) < y^*_{CK}(0)$.

Step 2. Pick a $\widehat{M} \in (\overline{M}, \infty)$. By Lemma E.10, $N \geq \widehat{N}(\gamma; \widehat{M})$ implies that $\underline{\alpha}^*_{CK}(N) \geq \frac{\widehat{M}}{N+1}$ since $\widehat{M}$ finite. By definition of $y^*_{CK}(N)$ (see (E.12)), it must be that for all $N \geq \widehat{N}(\gamma; \widehat{M})$: $(y^*_{CK}(N))^{\gamma-1} \leq \widehat{M}\left(1 - \frac{\widehat{M}}{N+1}\right)^N \frac{1-k_c}{2}$ since $\widehat{M} > \underline{M} \geq 1$ and consequently, $(1 - \underline{\alpha}^*_{CK}(N))^N \underline{\alpha}^*_{CK}(N) \leq \left(1 - \frac{\widehat{M}}{N+1}\right)^N \frac{\widehat{M}}{N+1}$. Notice that $\lim_{N\to\infty}\left(1 - \frac{\widehat{M}}{N+1}\right)^N = \exp(-\widehat{M})$ by L'Hospital's rule. Therefore, $\lim_{N\to\infty} y^*_{CK}(N) \leq \left(\widehat{M}\exp(-\widehat{M})\frac{1-k_c}{2}\right)^{\frac{1}{\gamma-1}} \equiv \overline{Y}(\widehat{M})$. Since by definition of $\widehat{M}$, $\overline{Y}(\widehat{M}) < y^*_{CK}(0)$, we must have $\lim_{N\to\infty} y^*_{CK}(N) < y^*_{CK}(0)$.

Step 3. Given that $y^*_{CK}(N)$ is continuous in N, $\lim_{N\to\infty} y^*_{CK}(N) < y^*_{CK}(0)$ implies that there exists a finite $N^y \geq \widehat{N}(\gamma; \widehat{M})$ such that for all $N \geq \widehat{N}(\gamma; \widehat{M})$, then $y^*_{CK}(N) \leq y^*_{CK}(0)$. □

Let $\underline{G}(0)$ and $\overline{G}(0)$ the equilibrium thresholds defined in the main text (see Proposition2).

Proposition E.3. *There exists a finite N^* such that for a non-empty open set of policy cost, for all $N > N^*$, then $\underline{G}(N) < \underline{G}(0)$ and $\overline{G}(N) < \overline{G}(0)$.*

Proof. Consider first a competent type. $\underline{G}(N)$ is implicitly defined by (E.27). By Lemma E.12, for all $N > N^y$, $C(y^*_{CK}(N)) < C(y^*_{CK}(0))$. By Lemma E.11, $\lim_{N\to\infty} \frac{(1-(1-\underline{\alpha}^*_{CK}(N))^{N+1}))}{2}(1-k_c) = \frac{1-k_c}{2} < \frac{\alpha^*_{CK}(0)}{2}(1-k_c)$ (since by Lemma 2 $\alpha^*_{CK}(0) < 1$). Therefore, there exists a finite $\breve{N}^*$ such that $\frac{(1-(1-\underline{\alpha}^*_{CK}(N))^{N+1}))}{2}(1-k_c) - C(y^*_{CK}(N)) > \frac{\alpha^*_{CK}(0)}{2}(1-k_c) - C(y^*_{CK}(0))$. Similarly, $\lim_{N\to\infty} \frac{1-q(1-(1-\underline{\alpha}^*_{CK}(N))^{N+1})}{2} = \frac{1-q}{2} < \frac{1-q\alpha^*_{CK}(0)}{2}$. Therefore, there exists a finite $\acute{N}^*$ such that $\frac{1-q(1-(1-\underline{\alpha}^*_{CK}(N))^{N+1})}{2}k_c > \frac{1-q\alpha^*_{CK}(0)}{2}k_c$. Using a similar reasoning as in Proposition E.2, there exists a finite $N^*(c)$ such that for all $N > N^*(c)$, $\underline{G}(N) < \underline{G}(0)$.

For a competent type, it can be checked using (E.10) that $\hat{y}_{CK}(N) < y^*_{CK}(N)$ for all N so $\hat{\alpha}_{CK}$ tends to 0 as N tends to infinity. Using (E.2), we get $(1-\hat{\alpha}_{CK}(N))^N = \left(\frac{\hat{y}_{CK}(N)}{y^*_{CK}(N)}\right)^{\gamma-1}(1-\alpha^*_{CK}(N))^N \frac{1-k_c}{1-k_n}$. This implies $\lim_{N\to\infty}(1-\hat{\alpha}_{CK}(N))^N = 0$ ($\lim_{N\to\infty}\left(\frac{\hat{y}_{CK}(N)}{y^*_{CK}(N)}\right)^{\gamma-1} = \infty$ would contradict $\hat{y}_{CK}(N) < y^*_{CK}(N)$). A similar reasoning as in Lemma E.11 yields then: $\lim_{N\to\infty}(1-\hat{\alpha}_{CK}(N))^{N+1} = 0$. By a similar reasoning as in Lemma E.12, there exists a finite $N^{\hat{y}}$ such that for all $N \geq N^{\hat{y}}$, $\hat{y}_{CK}(N) \leq \hat{y}_{CK}(0)$. A similar reasoning as above then implies that there exists a finite $N^*(n)$ such that for all $N > N^*(n)$, $\overline{G}(N) < \overline{G}(0)$. The claim holds for $N = \max\{N^*(c), N^*(n)\}$. □

The effect of heterogeneity To study the effect of heterogeneity in voters' policy payoffs, we introduce the following notation. Define $\underline{\Lambda} = \{\lambda_i = \underline{\lambda},\ \forall i \in \mathcal{N}\}$ and for all $j \in \mathcal{N} \setminus \{1\}$, define $\Lambda_j = \{\{\lambda_i\}_{i=1}^{N+1} : \lambda_j = \overline{\lambda} > \underline{\lambda},\ \lambda_k = \underline{\lambda},\ \forall k \neq j\}$. Denote also $A_i(\alpha^-_{CK}(\underline{\lambda}G; N, \Lambda_j)$ the probability of successful communication for the set of parameters $\{\Lambda_j, q, \gamma, G, N, k_c\}$ (see Lemma E.3 for a precise definition of $A_i(\alpha^-_{CK}(\underline{\lambda}G; N, \Lambda_j))$. As above, $\underline{\alpha}^*_{CK}(N)$ represents the probability of successful communication for the set of parameter $\{\underline{\Lambda}, q', \gamma, G, N, k_c\}$. The following lemma shows that introducing heterogeneity among voters can increase the probability of successful communication with all voters.

Lemma E.13. *For all* $N \geq 1$*, for all* $\overline{\lambda} \in (\underline{\lambda}, 1]$*, there exist* $q'(N), q''(N) \in (0, 1/2] \times [1/2, 1)$ *such that* $\underline{\alpha}^*_{CK}(N) < A_i(\alpha^-_{CK}(\underline{\lambda}G; N, \Lambda_j))$ *for all* $i, j \in \mathcal{N} \times \mathcal{N} \setminus \{1\}$*.*

Proof. We prove the result for $q'(N) \leq 1/2$, a similar reasoning holds for $q''(N) \geq 1/2$. The proof proceeds into steps. First, we show that $\alpha^-_{CK}(\cdot)$ is strictly increasing with λ_j when q is not too large. Second, we show that this implies the claim. For ease of exposition, we omit arguments whenever possible.

Step 1. The definition of $\alpha^-_{CK}(\cdot)$ (slightly abusing notation: $H_1(\alpha^-_{CK}(G_1); \lambda_j) = 0$) and the Implicit Function Theorem (given $\partial H_1(\alpha^-_{CK}(\cdot); \lambda_j)/\partial\alpha < 0$) imply that $\frac{\partial \alpha^-_{CK}(G_1)}{\partial \lambda_j}$ has the same sign as $\frac{\partial H_1(\alpha^-_{CK}(G_1); \lambda_j)}{\partial \lambda_j}$ (see (E.7)), that is:

$$\frac{\partial A_j(\alpha^-_{CK}(G_1))}{\partial \lambda_j} \prod_{l \neq 1, j}^{N+1} (1 - A_l(\alpha^-_{CK})) \begin{bmatrix} -\sum_{l=1}^{N+1} A_l(\alpha^-_{CK}) \prod_{h \neq l}^{N+1} (1 - A_h(\alpha^-_{CK})) + \\ +(1 - A_j(\alpha^-_{CK})) \prod_{h \neq j}^{N+1} (1 - A_h(\alpha^-_{CK})) + \\ -(1 - A_j(\alpha^-_{CK})) \sum_{l \neq j}^{N+1} A_l(\alpha^-_{CK}) \prod_{h \neq l, j}^{N+1} (1 - A_h(\alpha^-_{CK})) \end{bmatrix}$$

Since $\frac{\partial A_i(\alpha^-_{CK}(G_1))}{\partial \lambda_j} > 0$, $\frac{\partial H_1(\alpha^-_{CK}(G_1); \lambda_i)}{\partial \lambda_j}$ has the same sign as (after rearranging):

$$J(\alpha^-_{CK}) = (1 - 2A_j(\alpha^-_{CK})) \prod_{h \neq j}^{N+1} (1 - A_h(\alpha^-_{CK})) - 2(1 - A_j(\alpha^-_{CK})) \sum_{l \neq j}^{N+1} A_l(\alpha^-_{CK}) \prod_{h \neq l, j}^{N+1} (1 - A_h(\alpha^-_{CK})) \tag{E.29}$$

By the same argument as Lemma E.4, α^-_{CK} is strictly increasing in q (for $q < 1/2$) and such that $\lim_{q \to 0} \alpha^-_{CK} = 0$. Therefore, there exists $q^o(\lambda_j) \in (0, 1/2)$ such that for all $q \leq q^o(\lambda_j)$ $J(\alpha^-_{CK}) > 0$.

Step 2. Define $q'(N) \equiv q^o(\overline{\lambda})$ so $\alpha^-_{CK}(\cdot)$ is increasing with λ_j for all $\lambda_j \in [0, \overline{\lambda}]$. This implies that for all $N > 1$ and for all $q < q'(N)$, $\underline{\alpha}^*_{CK}(N) < \alpha^-_{CK}(\underline{\lambda}G; N, \Lambda_j)$ and $\underline{\alpha}^*_{CK}(N) < A_i(\alpha^-_{CK}(\underline{\lambda}G; N, \Lambda_j))$ for all $i \in \mathcal{N}$. □

The next proposition shows that introducing heterogeneity can mitigate the curse of the uninterested voter and exacerbate the curse of the interested voter. Denote $\underline{G}(N; \Lambda_j)$ and $\overline{G}(N; \underline{\Lambda})$ the equilibrium thresholds when the number of voters is N and the set of parameters is $\Lambda \in \{\Lambda_j, \underline{\Lambda}\}$.

Proposition E.4. *For all $N \geq 1$, $\overline{\lambda} \in (\underline{\lambda}, 1]$ and for all $q \notin (q'(N), q''(N))$ (defined in Lemma E.13), set the policy costs (k_c, k_n) such that $(\underline{G}(N; \Lambda_j), \overline{G}(N; \Lambda_j)) \in [0, 1]^2$. The equilibrium thresholds satisfy: $\underline{G}(N; \Lambda_j) < \underline{G}(N; \underline{\Lambda})$ and $\overline{G}(N; \Lambda_j) < \overline{G}(N; \underline{\Lambda})$*

Proof. Lemma E.13 implies that $(1-(1-\underline{\alpha}^*_{CK}(N))^{N+1}) < (1-\prod_{i=1}^{N+1}(1-A_i(\alpha^-_{CK}(\underline{\lambda}G; N, \Lambda_j))))$ for all $j \in \mathcal{N} \setminus \{1\}$. Using a similar reasoning as in Lemma E.5, $x^-_{CK}(G_i; N)$ is strictly increasing with λ_j for all $i \in \mathcal{N}$. The comparative statics on $x^-_{CK}(G_i; N)$ and Lemma E.13 then imply that an increase in λ_j relaxes the (IC) of a competent type (E.9) and tightens the (IC) of a non-competent type (E.11). This implies the claim by a similar reasoning as Proposition E.2. □